

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

Subject Name: Operations Research

Subject Code: 5SC01MTE1

Branch: M.Sc. (Mathematics)

Semester: 1

Date: 11/12/2015

Time: 10:30 To 01:30

Marks: 70

### Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
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### SECTION – I

Q-1 Attempt the Following questions.

(07)

a. Define: solution.

b. Define: Slack variable.

c. Which type of solution exist for given LPP

$$\text{Maximize } Z = 3x_1 + 4x_2$$

Subject to

$$x_1 - x_2 = -1$$

$$-x_1 + x_2 \leq 0$$

and  $x_1, x_2 \geq 0$

d. For minimization problem, what is coefficient of an artificial variable in the objective function?

e. The right hand side of the constraint in simplex method must be non-negative. Determine whether statement is true or false?

f. If dual has an unbounded solution, then primal has no feasible solution. Determine whether statement is true or false?

g. For maximization LP problem, the simplex method is terminated when all values  $z_j \leq 0$ . Determine whether statement is true or false?



- Q-2 Attempt all questions (14)**
- a.** A company sells two different products A and B, making a profit Rs.40 and Rs.30 per unit on them respectively. The production process has a total capacity of 30,000 man hours. It takes 3 hours to produce a unit of A and 1 hour to produce a unit of B. Maximum number of units of A can be sold is 8000 units and that of B is 12,000 units. Subject to these limitations products can be sold in market. Formulate this as a linear programming model to maximize profit. **(07)**

- b.** Write the dual of the following linear programming problem. **(07)**
- i) Minimize  $Z = 3x_1 - 2x_2 + 4x_3$       ii) Maximize  $Z = x_1 + 2x_2$   
 Subject to    Subject to
- |                             |                        |
|-----------------------------|------------------------|
| $3x_1 + 5x_2 + 4x_3 \geq 7$ | $2x_1 + 4x_2 \leq 160$ |
| $6x_1 + x_2 + 3x_3 \geq 4$  | $x_1 - x_2 = 30$       |
| $7x_1 - 2x_2 - x_3 \leq 10$ | $x_1 \geq 10$          |
| $x_1 - 2x_2 + 5x_3 \geq 3$  | and $x_1, x_2 \geq 0$  |
| $4x_1 + 7x_2 - 2x_3 \geq 2$ |                        |
| and $x_1, x_2, x_3 \geq 0$  |                        |

**OR**

- Q-2 Attempt all questions (14)**
- a.** Use the simplex method to solve the following LP problem **(07)**
- Maximize  $Z = 2x_1 + 5x_2$   
 Subject to
- |                       |
|-----------------------|
| $x_1 + 4x_2 \leq 24$  |
| $3x_1 + x_2 \leq 21$  |
| $x_1 + x_2 \leq 9$    |
| and $x_1, x_2 \geq 0$ |

- b.** What is Duality? What are the rules to form a dual problem from the primal problem? What are the advantages of Duality? **(07)**

- Q-3 Attempt all questions (14)**
- a.** Use the Big-M method to solve the following LP problem
- Minimize  $Z = 12x_1 + 20x_2$   
 Subject to
- |                         |
|-------------------------|
| $6x_1 + 8x_2 \geq 100$  |
| $7x_1 + 12x_2 \geq 120$ |
| and $x_1, x_2 \geq 0$ . |

- b.** Solve the following LP problem by graphical method **(07)**
- Maximize  $Z = 2x_1 + 3x_2$   
 Subject to
- |                     |
|---------------------|
| $x_1 + x_2 \leq 30$ |
| $x_2 \geq 3$        |
| $x_2 \leq 12$       |



$$x_1 - x_2 \geq 0$$

$$0 \leq x_1 \leq 20$$

OR

Q-3

**Attempt all questions**

- a. Solve the following LP problem by graphical method (07)

$$\text{Minimize } Z = 2x_1 + x_2$$

Subject to

$$x_1 - 3x_2 \leq 6$$

$$2x_1 + 4x_2 \geq 8$$

$$x_1 - 3x_2 \geq -6$$

$$\text{and } x_1, x_2 \geq 0.$$

- b. Use Two-Phase method to solve given LP problem (07)

$$\text{Minimize } Z = x_1 + x_2$$

Subject to

$$2x_1 + x_2 \geq 4$$

$$x_1 + 7x_2 \geq 7$$

$$\text{and } x_1, x_2 \geq 0.$$

## SECTION – II

Q-4

**Attempt the Following questions**

(07)

- a. Write Kuhn-Tucker condition for maximization type non-linear program.
- b. Determine, whether the function  $f(x) = 10 - x^2$  convex or concave?
- c. Define: Separable programming.
- d. Write Hessian matrix.
- e. What is unbalance assignment problem?
- f. What is a necessary and sufficient condition for the existence of a feasible solution to the transportation problem?
- g. If there were  $n$  workers and  $n$  jobs there would be  $n!$  Solution. Determine, whether statement is true or false?

Q-5

**Attempt all questions**

(14)

- a. Determine the initial basic feasible solution to the following transportation problem by using Vogel's Approximation method. (07)



Plant	Distribution Centre				Supply
	$D_1$	$D_2$	$D_3$	$D_4$	
$P_1$	1	3	1	4	30
$P_2$	3	3	2	1	50
$P_3$	4	2	5	9	20
Demand	20	40	30	10	100

- b. A company has four machine to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table: (07)

		Machine			
		W	X	Y	Z
Job	A	18	24	28	32
	B	8	13	17	18
	C	10	15	19	22

What are the job assignments which will minimize the cost?

OR

Q-5

Attempt all questions

- a. Find optimum solution by MODI method. If alternate solution exist then find. (07)

	$D_1$	$D_2$	$D_3$	supply
$S_1$	4	8	8	76
$S_2$	16	24	16	82
$S_3$	8	16	24	77
Demand	72	102	41	

- b. Use the dual simplex method to solve the LPP (07)

Maximize  $Z = -2x_1 - x_3$

Subject to

$$x_1 + x_2 - x_3 \geq 5$$

$$x_1 - 2x_2 + 4x_3 \geq 8$$

and  $x_1, x_2, x_3 \geq 0$ .

Q-6

Attempt all questions

- a. Solve the following non-linear programming graphically (14)

Minimize  $Z = x_1^2 + x_2^2$  (07)

Subject to

$$x_1 + x_2 \geq 8$$

$$x_1 + 2x_2 \geq 10$$

$$2x_1 + x_2 \geq 10$$

and  $x_1, x_2 \geq 0$

- b. Solve the following non-linear programming problem by Lagrangian multiplier (07)



method

$$\text{Maximize } Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$$

Subject to

$$x_1 + x_2 = 2$$

and  $x_1, x_2 \geq 0$

**OR**

**Q-6**

**Attempt all Questions**

- a. Determine the relative maximum and minimum of the following function (07)

$$f(X) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2 .$$

- b. Use the wolfe's method to solve the quadratic programming problem. (07)

$$\text{Maximize } Z = 2x_1 + x_2 - x_1^2$$

Subject to

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

and  $x_1, x_2 \geq 0$ .

